**EXPLORING THE HETEROGENEITY** **INSIDE POPULATION**

**TO ENHANCE CROWD MODELLING OF GROUP DYNAMICS**

1. **Introduction**

Rapid urbanization and population growth are always inevitable challenges for every country in the effort of planning infrastructure, estimating traffic needs and capacities, and increasing the safety of pedestrians since over 70% of the world population is predicted to live in cities by 2050 (Weidmann, 2012). With the increase in the number of public events and the number of accidents during these events (Evers, 2011), the demand for realistic crowd simulation models becomes important for risk management in urban design and crowd safety. To develop realistic simulation models, various studies have been conducted in understanding and simulating behaviours which can unfold in both normal and emergency situations such as groups of pedestrians moving with or compete against each other.

Group cohesion behaviour is the behaviour of objects moving towards the average positions of their neighbours over the time (Reynolds, 1987). The definition of this behaviour was motivated by the visual appearance of coherently flying objects. The behaviour has been then investigated widely on the collective motion of different flocking organisms including homing pigeon flocks (Kattas, 2012) (Nagy & Vicsek, 2010), fish schools (Couzin, 2013), and bacteria (Cisneros, 2007).

Human group cohesion behaviour is observed by its cohesion degree and formation. Cohesion degree denotes the average distance to the group’s centre of mass from each group member while observable human group formations are V-like, line-abreast, U-like, or river-like (Helbing, 2005). Group cohesion behaviour is important in both normal and evacuation scenarios. In normal situations, group cohesion behaviour can affect the speed and movement direction of pedestrians who are not belonging to any group. In human behaviour research, group cohesion behaviour has been observed in the UK at different places of train station, shopping centre, university campus, Clumber Street and showed that pedestrians in crowd are in group of two or more members. The percentages of people in groups at these places are 37%, 50%, 28%, 50% respectively (Singh, 2009). Pedestrians in the same group might be family members, colleagues. In crowd disasters, pedestrians belonging to same group may move irrationally to maintain its cohesion and consequently become obstacles for other pedestrians (Aguirre, 2011).

Various models have been proposed to simulate group cohesion behaviour such as the social-force based model, the cellular automata model, the standard Vicsek model. These models mainly focusing on creating various behaviour rules to explain how groups move in evacuation scenarios or investigating how group’s formation and cohesion degree change when group population size varies, exploring collective behaviour of flocking organisms together with heterogeneity of group members through generating differently external noises. However, these models have not yet explored the impact of the agent’s initial parameters on group cohesion behaviour, even though an actual group contains different group members, whose individual physical parameters (speed, interaction range and strength) are different to those of others. Groups of different members can be easily seen in both of normal and emergency situations. A study, (Aguirre, 2011) found that a pedestrian may select another pedestrian based on demographic traits to move together in a group through the crush disaster that happened at the Nightclub, USA in 2003.

Therefore, this PhD study aims to investigate the effect of the parameters of individual group members on the group cohesion behaviour and thus on flow rates of group movement. This impact is then investigated through proposed case studies of simulation scenarios.

Section 2 of this report represents the state of the art from models of group cohesion behaviour and their advantages and limitations. Section 3 analyses the drawbacks of current models and presents the need for this research study. Section 4 presents proposed research questions. Section 5 reports the contribution of this study. Section 6 presents research methodology to achieve these questions. Section 7 reports current work in progress for answering these questions. Finally, section 8 outlines compulsory research training hours undertaken in the IT faculty.

1. **Literature Review**

This section reviews current models that incorporate group behaviour. Modelling approaches are various from investigating social forces that affect each pedestrian’s acceleration, modelling the changes of each cell on a grid layout, to defining behaviour rules to describe agents follow other, and providing standard Vicsek model which has been applied widely in flocking organisms.

**2.1. Social force model for group behaviour**

Moussaid, Helbing and colleagues (Moussaid, 2010) created the social group model based on the social-force model (Helbing & Molnar, 1995, 2000). The social group model (equation 1-2) represents that a pedestrian *p* at time *t* is trying to move with a certain desired speed in a desired direction pointing from pedestrian *p*’s current position to his target position. Therefore, pedestrian *p* tends to correspondingly adapt his actual velocity with a certain acceleration time . The acceleration time represents pedestrian *p* changes its current velocity and return to its desired velocity. Pedestrian *p*’s acceleration at time *t* is also influenced by repulsive forces coming from surrounding pedestrians and obstacles. They are and respectively. The repulsive force’s directions and group force direction are represented in Figure 1. The group influence force aims to describe that an individual in group continuously adjusts its position to reduce its head direction and maintain group’s centre of mass, but also avoid other group members. The group force is represented in equation 3.

|  |  |
| --- | --- |
|  | (1) |
| = | (2) |
|  |  |

where is the desired speed of pedestrian *p* that varies over time, is an uncertainty factor.

|  |  |
| --- | --- |
|  | (3) |



**Fig 1**. Repulsive forces and on pedestrian *p* created by pedestrian *q* and wall γ.

The social group force describes that pedestrian p at time *t* turns his gazing direction to see their partners. Thus, vision force is included to help pedestrian p adjust its position to reduce the head rotation. At the same time, pedestrian p keeps a certain distance to the group’s centre of mass by the force . A repulsive force is added to support pedestrian *p* avoid other group members.

To summary, the social force model comprises parameters that need to be set at initial simulation time as in Table 1:

**Table 1** – Social-group force model’s parameters

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Component** | **Description** |
|  | Desired Acceleration | Initial desired velocity |
|  | Desired Acceleration | Acceleration time to reach desired speed |
|  | Desired Acceleration | Constant to find maximum velocity |
|  | Repulsive Force with other pedestrians | Angular component |
| *A* | Repulsive Force with other pedestrians | Interaction strength |
| *B* | Repulsive Force with other pedestrians | Interaction range based on distance between *p*, *q* |
| U | Obstacle Force | Obstacle interaction strength |
|  | Simulation | Radii of pedestrian *p* in simulation environment |
|  | Group vision force | The strength of the social interactions between group members |
|  | Group attraction force | The strength of the attraction effects |
|  | Group repulsion force | The repulsion strength between group members |

Social-force based model has possessed a long-life modification period by its author and colleagues for more than a decade in order for simulating the additional factors affecting individual’s acceleration or being easier towards calibration process. However, it almost uses the same parameter distribution to simulate pedestrians inside crowd as in Table 2.

**Table 2** – Social-group force model’s parameter value

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Value** | **Reference** |
| (m/s) | avg. = 1.34, st. dev. = 0.26 | (Helbing, 1995) |
| avg. = 1.3, st. dev. = 0.3 | (Helbing, 2005) |
| (s) | 0.5 | (Helbing, 1995) |
| 1.0 | (Helbing, 2000), (Helbing, 2005) |
|  | 1.3 | (Helbing, 1995), (Helbing, 2005) |
| *A* (m/s2) | 3.0 | (Helbing, 2005) |
| *B* (m) | 0.2 | (Helbing, 2005) |
|  | 0.75 | (Helbing, 2005) |

Through observation, Moussaid found that pedestrians in same group likely move in a line-abreast formation to allow them communicate with each other easily. When crowd density increases, group of pedestrians automatically change its formation into V-shaped or river-like pattern. According to the study, when the model parameter = 0, it shows that group members only try to stick together with no communication rule. When = 4, a V-shaped structure is created.

The authors applied the same value of each parameter in Table 2 and parameters of group force including to all pedestrians inside group to see these patterns. In fact, human group formation is various from V-line, U-like, line-abreast, to river-abreast as in actual observation (Helbing, 2005). However, this model did not mention at which values of parameters other group formations could be created. It also raises a question whether these parameters have to be the same for all group members to establish these structures.

* 1. **Cellular automata model for group behaviour**

Cellular automata-based group behaviour model is the approach relying on of Von Neumann’s idea that divides space into uniform grid or hexagonal cells. At each time *t*, variables at each cell are updated according to a set of local rules or its neighbour cells (Zheng, 2009). Common local rules are moving direction, or avoidance rules. Every cell in the space can be in different states including free, an obstacle, or occupied by a pedestrian. General cellular automate model is formed as formulas 4-6.

|  |  |
| --- | --- |
| where | (4) |
|  | (5) |
|  | (6) |

Every cell has variables of path field, obstacle field, and density field. Path field is to identify distance from current cell to destination cell. Obstacle field indicates for every cell the distance from an obstacle or a wall. Density field is to indicate for each cell the crowd density in the surroundings at the current time step *t.* When running a CA-based pedestrian model, there is several update strategies including parallel update, sequential update, or shuffled sequential update.

To simulate group behaviour, Vizzari (Vizzari, 2013) constructed pedestrians on these defined cells. A pedestrian is represented as a utility-based agent having following attributes:

|  |  |
| --- | --- |
|  | (7) |

where:

* Id: identification number of pedestrian *i*
* GroupId: identification number of group that pedestrian *i* belongs to
* State: represents pedestrian’s current cell that and direction followed in last movement
* Actions: is the set of possible actions to choose an appropriate cell from equation (5) and equation (6).
* Destination: reflects current path field of the cell where pedestrian *i* is in

A utility function was proposed by the author as in equation 8. The function estimates the probability of cell c to allow pedestrian *i* move in to maintain group cohesion at each time step *t*.

|  |  |
| --- | --- |
|  | (8) |

where:

* , , , , , , are model’s parameters for their corresponding functions
* is the goal attraction derived from current cell’s path field and destination cell’s path field
* represents obstacle repulsion from obstacle field of current cell *c* over the maximum distance to obstacles from any cell in grid layout
* represents separation value to allow pedestrian *i* avoid other pedestrians. It is measured by density field of current cell *c* over the predefined maximum density.
* represents whether this cell is the same direction with previous movement of pedestrian
* represents a small probability to allow two pedestrians stay on the same cell.
* represents cohesion value of cell *c* if pedestrian *i* move in towards other group member’s position
* is used in the case of large group which can be separated into sub groups. It represents the cohesion value of current pedestrian toward the largest group.
* is the distance from cell *c* to pedestrian *i*’s current cell position. *d* is only equal to 1 or

Group cohesion degree is then defined as in equation (9) to represent the average distance from each group member to group’s centre of mass. The study used this degree to support pedestrian *i* trade off current goal attraction with group cohesion based on predefined rules.

|  |  |
| --- | --- |
|  | (9) |

The study then measured the correlation between group size and speed in various design layouts. However, this CA-based model only allows pedestrians move in neighbour cells rather than in further cells at each time step. Moreover, it applied the same value of each parameter, , , , , , for whole group members to measure the group speed. Thus, it neglected the heterogeneity in speed, interaction strength, and model parameters of actual group members. The effect of these parameters on group formation was not investigated.

**2.3. Agent-based model for group behaviour**

In agent-based model, (Pelechino, 2006) constructed a simulation environment and created different pedestrian roles (leader, untrained leader, group members) through agent-based model to simulate evacuation scenarios. (Aguirre, 2011) constructed a simulation environment and compared the difference in escape numbers of several prototypes constructed on agent-based model. The prototypes include individual behaviour, intermediate group (revert to individual behaviour while in duress), full group behaviour (follow group leader). The escape numbers are compared with actual survivor number. On social aspect, the author mentioned that a group leader can be selected by other through demographic traits such as age, gender and familiarity with environment. A group member follows leader if they are in the leader’s line of sight.

Another recent study (Cheng, 2014) constructed an agent-based model to simulate passengers at airport-check in area. The result shows that group behaviour has a significant impact on the performance and utilisation of airport’s services. The study also presents a case study of the model to evaluate evacuation strategies of the airport. By comparing the evacuation times of individual passengers and group of passengers, the study found that group dynamics influence significantly on evacuation behaviour of passengers and total evacuation time.

However, these models did not investigate group cohesion degree and how group behaviour affects flow rate when varying parameters of group members. They only focus at creating variously rules for pedestrians follow other to measure escape rate.

**2.4. Standard Vicsek Model for cohesion behaviour of flocking organisms**

In order to interpret the behaviour of huge flocks of living organisms (flock of birds, fish schools, and bacterium, and human crowd) in the presence of perturbations, a statistical physic approach has been introduced to the flocking by Vicsek (Vicsek, 1995). Nowadays, it has been called as Standard Vicsek Model as suggestion of (Huepe & Aldana, 2008) (Bertin, 2009). The model considers that self-propelled particles represent living flocks, and perturbations are natural consequence of stochastic and deterministic factors affecting the motion of particle. The model is presented in equations 10-11.

|  |  |
| --- | --- |
| + *pertubations* | (10) |
|  | (11) |

The main idea of the model is that at each given time step *t*, particle *i* is usually controlled by interactions with its local neighbours in a constant radius *R* and uncertainty factor perturbations.

Here denotes the averaging of the velocities of neighbours in radius *R*. The expression provides a unit vector pointing in the average direction of motion. The particle *i* also has a constant velocity . In the standard version of the model, Vicsek derived the perturbations factor by adding a random angle to the angle corresponding to the average motion direction of particle i’s neighbourhood. The angel of average motion direction and random angle at time *t* are represented as in equations 12-13.

|  |  |
| --- | --- |
|  | (12) |
|  | (13) |

where and are the x and y coordinates of particle jth’s velocity in the neighbourhood of particle *i*. The perturbation is a random number taken from uniform distribution in the interval [ ]. The randomness of perturbation makes particles have different motion direction from those of others. The velocity was set the same for all birds in flocks. Finally, two control parameters of the model are the density (number of particles in a volume (d is the dimension)), and the level of perturbation (.

In a study of the authors (Crizok & Vicsek, 2006), the average momentum of the particles and the correlation between particles’ velocity directions were investigated when varying model’s parameters including the level of perturbation, the density , and population size.

There is also another approach from the author to investigate the role of model’s parameters (Bhattacharya & Vicsek, 2006) on group cohesion behaviour. This study derived the model in 3D dimensional environment to explore the cohesiveness through the process of landing of bird flocks performing foraging flights. The study explored the heterogeneity in attributes such as the ages, sex, and social status of animals in group or the differences in the perception of external stimuli by assigning to each bird *i* an inherent switching time , such that if the bird begins a flight at time *t*=0, it would decide to land at time t= .This work was to show that the difference in the attributes implied the difference in energy reserve to maintain an altitude. ’s was selected from a Gaussian distribution with a given standard deviation . The study then investigated quantitatively the fraction of birds not landed yet as time *t* progresses when setting to different values. However, the model’s parameters , , were set the same for all birds.( = 2.0, , *R*=2.0, *v*= 0.01).

In summary, standard Vicsek model used the particle-based approach to understand flocking organisms. The author’s proposed studies investigated collective behaviour when varying external noise over the time, adding a new constraint for landing period of individual group members, or varying model’s parameters. However, these studies have yet explored collective behaviour when considering individual group members have different parameter distributions in these parameters.

1. **Problem Statement**

Human group cohesion behaviour modelling is important since it helps to simulate correctly how pedestrians in a group automatically changes its group formation to adapt to different situations and represents the effect of the group for escape rate measurement. In a recent study (Aguirre, 2011), it found that modelling with group cohesion behaviour could generate more accurate survivor numbers than the model only simulating individual behaviour.

Through literature review in section 2, the models mainly categorized into three approaches including agent-base models, the force-based model, and the cellular automata model, and the standard Vicsek model.

In agent-based approach, various behaviour rules describing how pedestrians follow together have been created and the difference of survivor numbers between simulation prototypes using different rules are compared. However, one of the clearest limitations of agent-based models mentioned in the future work of the study (Weijmen, 2013) is the lack of a standard mechanism to measure the effects of agent’s parameters in the pedestrian’s force calculation. Moreover, agent-based approach only focuses on defining rules rather than investigating the impact when varying agent’s parameters on escape rate.

Current group force model and cellular automata model make assumption that populations are homogeneous. According to parameters of social force model (parameters in Tables 1 and 2, and parameter of group force ) and automata model (parameters in equation 8), these authors set the same parameter values for all pedestrians. These two models almost investigate group speed, group formation, and group cohesion degree when varying group population size. Considering group members are homogeneous is not true for an actual group which contains different members in age whose physical attributes such as speed, interaction strength are different (Daamen & Hoogendoorn, 2012). Thus, this limitation makes modellers and simulation software’s end users simulate inaccurately different group members. It raises a question about whether these group-force parameters are different for every group members to help them maintain group cohesion.

Standard Vicsek model relies on particle-based approach to simulate the cohesiveness of flocking organisms. Vicsek and colleagues explore average momentum of flocks and velocity correlation of group members when model’s control parameters (interaction radius, random noise) vary. However, these studies consider that individual group members have the same control parameters. They have not yet explored the behaviour of flock when individual group members have different parameter distribution to those of others.

To summary, the effect of group member’s parameters has not been investigated on group cohesion degree and flow rate through interacting with other group members and out-group pedestrians respectively.

**4. Research Question**

Current group cohesion models have not yet explored the impact of the agent’s parameters on group cohesion behaviour, even though an actual group contains different group members, whose individual parameter distributions are different to those of others. Thus, this PhD research aims to explore the heterogeneity of group members’ initial parameters in these models by providing questions.

1. What is the effect of the initial parameters of individual group member in group cohesion models in which group member types have different parameter distributions?

* 1. What is the difference in flow rates between setting different parameter distributions and averaging out parameter distribution for simulating a crowd of different pedestrian types?
  2. How do each group member’s parameters affect flow rate when group members interact with out-group pedestrians?
  3. How does group cohesion degree change when the group member’s parameters vary?

**5. Research project’s contribution**

Group cohesion is important since it affects flow rate measurement in both of evacuation scenario and the scenarios of the same or oppositely moving pedestrians along a corridor. Current models make pedestrians homogeneous and use the same parameters to explore the effect of group cohesion on group speed and flow rate. They have not investigated the difference in group members’ parameter distribution. Thus, the provided study will provide a detailed understanding of the impact of individual group member’s parameters on group cohesion models. It will enable:

* Modellers understand possible impacts of group cohesion behaviour on flow rates according to different parameter settings of group members when testing different design layouts.
* Event organizers can assess different evacuation strategies in simulate environment via flow rate measurement when simulating with the group behaviour.
* Event organizers restore the order of crowd before deteriorative situations can occur when groups of different members are trying to escape a door in live events.
* Event organizers predict the change of space occupation of group when group cohesion degree varies in real time.

1. **Research methodology**

This section presents the research methodology to resolve the proposed questions. The main question is to explore the impact of group member’s parameters on flow rate and group cohesion degree when group members maintain cohesion behaviour.

* 1. What is the difference in flow rates between setting different parameter distributions and averaging out parameter distribution for simulating a crowd of different pedestrian types?

Since social-force model and group-force model keep the same parameter values for group members, first work in this question is to investigate whether setting different parameter distributions and averaging out parameters distributions for pedestrian types generate the same flow rates. According to a recent calibration study (Daamen & Hoorgedoorn, 2012), it found that different pedestrians, who are different in age groups (children: <14 years old, adults, elders > 60 years old), are different in value distribution of each parameter (desired acceleration, acceleration time, interaction strength, and interaction range) as in equations 10-13. Thus, a hypothesis testing is applied in the original force model to measure flow rates of two prototypes. The first prototype uses the same parameter distribution and the second prototype uses different parameter distributions for pedestrian types. Escape rates of two prototypes are investigated through hypothesis testing and blockage frequency measurement.

|  |  |
| --- | --- |
|  | (10) |
|  | (11) |
|  | (12) |
|  | (13) |

After measuring two approaches, group force model is simulated to measure the impact of each group member’s parameter settings.

* 1. How do each group member’s parameters affect flow rate when group members interact with out-group pedestrians?

This sub question considers a group of pedestrians (called as group *A*) are moving towards a defined goal and maintaining group’s centre of mass, and another crowd (called as crowd *B*) of pedestrians moving individually towards the goal without influenced by social group force. This sub question considers two scenario types. The first type is evacuation scenario. The last one is the scenario of moving pedestrians along a corridor.

This sub question aims to understand the effect of how a group becomes obstacle for out-group pedestrians when it maintains group cohesion based on group member’s parameters. Figure 2 summarized proposed methodology for this work.

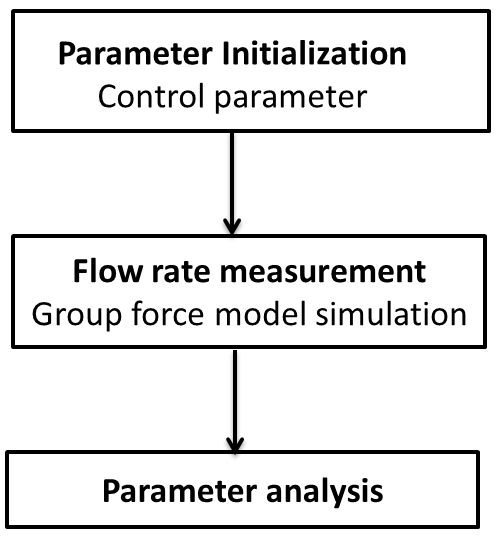


Figure 2. The architecture of investigating group member’s parameter on flow rate

Parameter initialization phase aims to generate initial parameter values for the different group member types in group *A* on the parameters of desired acceleration, acceleration time, interaction strength, and interaction range. This phase only samples at the parameter or the combination of multiple parameters being investigated. At each parameter control, different parameter distributions of group member types are generated from a base distribution of a specific group member type as in equation 14-15. This work is to reduce the complication for handling the control parameter.

|  |  |
| --- | --- |
|  | (14) |
|  | (15) |

where , represent distribution of the control parameter *c* on specific group member type, and , , are model’s coefficient parameters

Social group force model is then simulated to measure flow rates in different simulation environments of evacuation scenario and flow motions. Each simulation environment will also consider both of deterministic model or non-deterministic model by adding random noise taken from a uniform distribution. With the control parameters and their distributions identified in previous phase, each simulation is sampled repeatedly on the distributions.

Parameter analysis phase investigates whether individual control parameter affect flow rate and the interaction of control parameters on flow rate based on the analysis of variance (ANOVA). The change of flow rates is also investigated when varying these control parameters to determine areas in which flow rates change smoothly or discontinuous.

* 1. How does group cohesion degree change when the group member’s parameters vary?

By applying the architecture of investigating group member’s parameters denoted in Figure 2, this sub question aims to understand how spatial occupation of groups vary when groups interact with out-group pedestrians.

//need table

In social group force model (Moussaid & Helbing, 2010) represented in literature review, the authors haven’t yet explored the role of group force parameters on individual group members. Thus this question aims at understanding the impact of these parameters of each group member type on group cohesion degree.

* What is the impact of group force parameters of each group member type on group cohesion degree?

For each group member type, a scanning parameter space is performed on each group force parameter to understand at which values, following criteria is emerged:

* a new pattern of group cohesion degree is established
* group cohesion degree changes smoothly or discontinuous

It aims to determine areas having the same criteria on two-dimensional space of each pair of group force parameters when varying parameter values in horizontal and vertical directions.

The relationship between each group force parameter and group cohesion degree is investigated through ANCOVA approach. The interaction effect of each pair of group force parameters on group cohesion degree is also investigated through ANOVA.

* How does group cohesion degree change when new group members join or split group?

The combination of three parameters of each group member type is also investigated on group cohesion degree in below scenarios:

* the changes of group cohesion degree when removing *k* agents in group
* the changes of group cohesion degree when new *n* agents join group

1. **Research progress**
   1. Question 1

Simulation environment

Hypothesis testing of two prototypes about escape rates and blockage frequencies

* 1. Group force model simulation
  2. Research time line

1. **Coursework and professional development**

As required from our faculty, I completed the course FIT 5143 in the first semester 2015. I am attending the course FIT6021 from 21 July, 2015. I also completed 116 research training hours as in Table 3.

**Table 5**- List of professional development undertaken

|  |  |
| --- | --- |
| **Activity** | **Hours counted towards coursework goal** |
| Faculty Induction | 4 |
| Research Integrity | 12 |
| FIT 5143 Course | 48 |
| FIT 6021 |  |
| FIT 4012 | 15 |
| Monash Seminar/workshop attendance | 22 |
| Participation at Monash Bootcamp Commercialisation workshop in the year 2015 | 15 |

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